

Hypercentral Constituent Quark Model with a Meson Cloud

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Abstract

The results for the elastic nucleon form factors and the electromagnetic transition amplitudes to the $\Delta(1232)$ resonance, obtained with the Hypercentral Constituent Quark Model with the inclusion of a meson cloud correction are briefly presented. The pion cloud effects are explicitly discussed.

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Key words: Electromagnetic form factors, nucleon and $\Delta(1232)$, Hypercentral Constituent Quark Model, meson cloud.

1 Introduction

In recent years, various constituent quark potential models have been proposed to describe the intrinsic structure of baryons. Typical examples of the models are the Isgur-Karl model [1], the Capstick and Isgur relativized model [2], the chiral model [3] and the Hypercentral Constituent Quark Model (HCQM) [4]. Various baryon properties, such as baryon spectroscopy, nucleon form factors and the transverse and the longitudinal electromagnetic transition amplitudes of 3 and 4 stars resonances, have been systematically calculated with the HCQM [4–6]. In this work, we will present calculations of the nucleon electromagnetic form factors and nucleon- $\Delta(1232)$ electromagnetic transition amplitudes based on the framework of the Hypercentral Constituent Quark Model with a meson cloud. Particularly, we shall stress the corrections due to the pion meson cloud. In our calculation, a baryon is considered as a three-quark core surrounded by the pion meson cloud. Thus, we have new degrees of freedom in addition to the conventional three constituent quarks.

2 The Hypercentral Model

The Hypercentral Constituent Quark Model [4] contains a linear plus a coulomb-like potential

$$V(x) = -\frac{\tau}{x} + \alpha x, \quad \text{with} \quad x = \sqrt{\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2} \quad (1)$$

where x is the hyperradius defined in terms of the standard Jacobi coordinates $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$. It should be mentioned that this hypercentral potential has the following features. First of all, it contains three-body force effects. Secondly, it can be considered as the hypercentral approximation of a two-body potential of the form linear plus coulomb, as suggested by lattice QCD calculations [7]. Thirdly, its predictions of the proton form factors decrease as powers of the virtual photon momentum, while the form factors of the conventional harmonic oscillator potential decrease as gaussians. Finally, its predictions for the transition amplitudes $A_{1/2}(Q^2)$ of $S_{11}(1535)$ and $D_{13}(1520)$ resonances [5] agree with the data, particularly in the momentum transfer region of $Q^2 \sim 1 \sim 1.5 \text{ GeV}^2$.

The hypercentral model contains also a standard hyperfine interaction fitted to the $N - \Delta$ mass difference and in this form it has been employed for the calculation of various baryon properties [5,6,8]. However it has been shown that a good description of the lower part of the spectrum and of the transition amplitudes can be obtained also using simply the hyper-Coulomb (Hyc) potential to which a linear and a spin-dependent interactions are added as perturbations [9,10].

Thus, for the low-lying resonance states it is a good approximation to simply take the space wave functions of the Hyc potential instead of the ones coming from the numerical solution of the linear plus Coulomb potentials with hyperfine interaction.

3 Pion meson cloud correction

To take the pion meson cloud effects into account, one may consider the following Lagrangian density with a πqq coupling (similar to the πNN case) [11]

$$\begin{aligned} \mathcal{L} = & i\bar{\psi}_q(x)\gamma^\mu\partial_\mu\psi_q(x) - m_q\bar{\psi}_q(x)\psi_q(x) + \frac{1}{2}(\partial_\mu\pi(x))^2 - \frac{1}{2}m_\pi^2\pi^2(x) \\ & - ig\bar{\psi}_q(x)\gamma_5\psi_q(x)\tau \cdot \pi(x). \end{aligned} \quad (2)$$

From Eq. (2) the conserved local electromagnetic current can be derived using the principle of minimal coupling $\partial_\mu \rightarrow \partial_\mu + ie_q A_\mu$, where e_q is the charge carried by the field upon which the derivative operator acts. The total electromagnetic current J^μ is then

$$J^\mu(x) = j_q^\mu(x) + j_\pi^\mu(x), \quad (3)$$

where

$$\begin{aligned} j_q^\mu(x) &= \sum_a Q_a e \bar{\psi}_a(x) \gamma^\mu \psi_a(x), \\ j_\pi^\mu(x) &= -ie[\pi^\dagger(x)\partial^\mu\pi(x) - \pi\partial^\mu\pi^\dagger(x)]. \end{aligned} \quad (4)$$

Because of the πqq coupling, a physical baryon state can be described as a superposition of a three-quark core and its surrounding pion cloud,

$$|A\rangle = \sqrt{Z_2^A} \left[1 + (E_A - H_0 - \Lambda \mathcal{H}_{int} \Lambda)^{-1} \mathcal{H}_{int} \right] |A_0\rangle, \quad (5)$$

where Z_2^A is the bare baryon probability in the physical baryon state, Λ is the projection operator projecting out all the components of $|A\rangle$ with at least one pion, and \mathcal{H}_{int} is the interaction Hamiltonian which describes the process of emission and absorption of pions, which can be obtained from the Lagrangian density with the πqq coupling, Eq. (2) [11].

Our numerical calculations of the elastic nucleon form factors and of the $N - \Delta$ electromagnetic transition amplitudes are performed with the analytical model of Ref. [9, 10] without the spin-spin interaction and with the explicit inclusion of the pion cloud corrections. We set the parameters for the HCQM potential as $\tau = 4.59$, $\alpha = 1.61 fm^{-2}$ [4] and $g = 0.585$ (corresponding to the usual πNN coupling constant $g_{\pi NN}^2/(4\pi) = 13.6$).

Moreover, we also take the results of the conventional harmonic oscillator potential with $\alpha = 0.410 GeV$ (corresponding to a wave function with radius of the order of $0.5 fm$) for a comparison. To calculate the electromagnetic interaction between the photon and the nucleon with the pion meson cloud, we consider the following couplings: photon-quark, photon-quark with the pion meson cloud in flight, and photon-charged pion. Fig. 1 illustrates the three couplings between the photon and the nucleon or the Δ with the pion meson cloud.

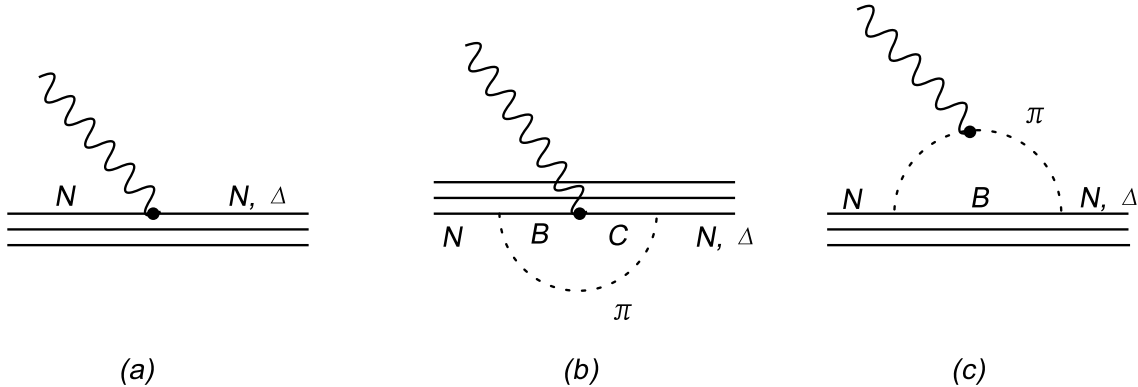


Fig. 1: Diagrams illustrating the various contributions included in the calculation. The intermediate baryons B and C are here restricted to the N and Δ .

4 Results and discussions

As shown in Eq. 5, a baryon wave function contains a three-quark core component and a three-quark core plus a pion meson cloud component. One can directly calculate the three-quark core probability Z_2^A [11]. For the nucleon, we have $Z_2^N = 0.567$ in the Hyc potential and $Z_2^N = 0.442$ in the classical HO potential. Our result for $\langle r \rangle_P$ is $0.68 fm$ in Hyc potential and $0.54 fm$ in the HO potential. The probability of the pion meson in

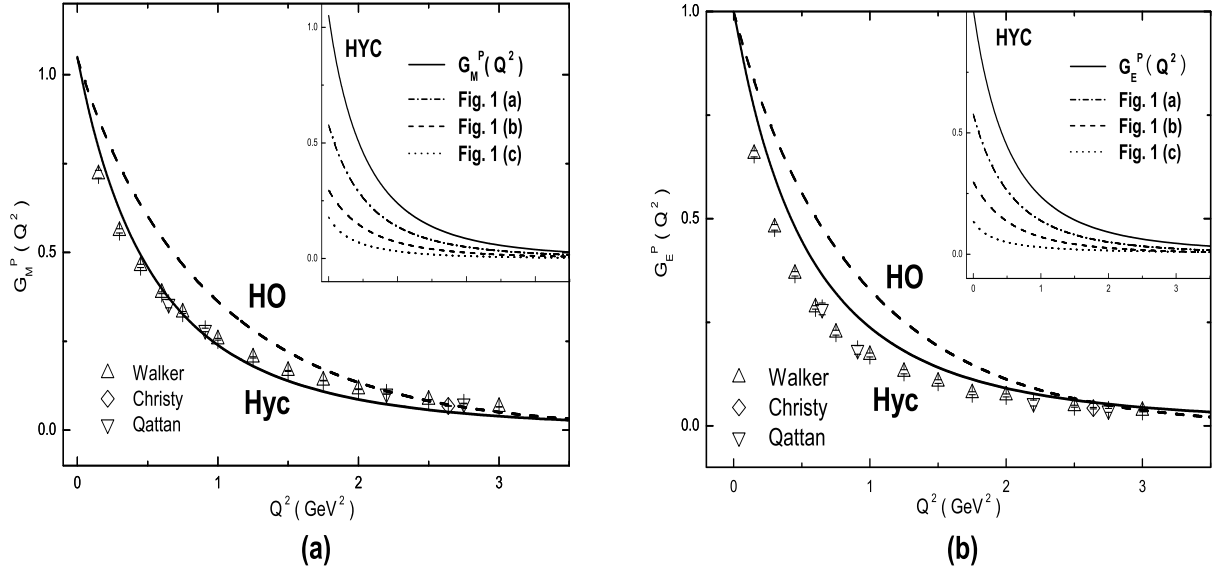


Fig. 2: Comparison between the experimental data and the theoretical predictions (Hyc and HO) for (a) the proton magnetic form factor and (b) the proton electric form factor. The experimental data are taken from [12]. Each figure also explicitly shows the different contributions from Fig. 1 for the Hyc case.

the HO potential is larger than that in the Hyc potential. This can be explained by the fact that the pionic contribution is competing with that of the quark core and a smaller r.m.s radius means a stronger pion coupling.

Figure 2 shows the obtained proton electromagnetic form factors compared with the experimental data. We use the Hyc wave function as the wave function of the nucleon. Figure 2 also shows different contributions from Fig. 1 to the proton form factors (see the insets in the top right corner of Figs.(2a) and (2b)). In the small Q^2 region, the contribution of the $\gamma\pi\pi$ interaction is more than 15% and as Q^2 increases it decreases quickly. When Q^2 is beyond 1.0 GeV^2 the contribution nearly vanishes. One finds that the contribution of the pion meson cloud is important especially in the low Q^2 region and also one can draw the conclusion that for the proton form factors the hypercentral constituent quark model can give a better prediction than that of the HO model.

Our results for the neutron electromagnetic form factors are shown in Figure 3. For the magnetic form factor the Hyc gives a better prediction, however for the electric form factor the result of the Hyc wave function is smaller than the experimental data. It is also smaller than that of HO potential since the pionic contribution in the Hyc potential is smaller and the non-zero value of the neutron charge distribution in the present calculation results from the pionic contribution. It should be stressed that here we have not considered the hyperfine mixing. We know that spin-spin forces lead to configuration mixing; for example, the octet wave function $\psi(8, \frac{1}{2}^+)$ not only gets contribution from the octet $|56, 0^+\rangle_{N=0}$ but also from $|56, 0^+\rangle_{N=2}$, $|70, 0^+\rangle_{N=0}$ and $|70, 2^+\rangle_{N=2}$. This mixing gives rise to a non-zero electric form factor of the neutron. It is expected that the mixing effect has to be taken into account for a further analysis of the neutron charge form factor in

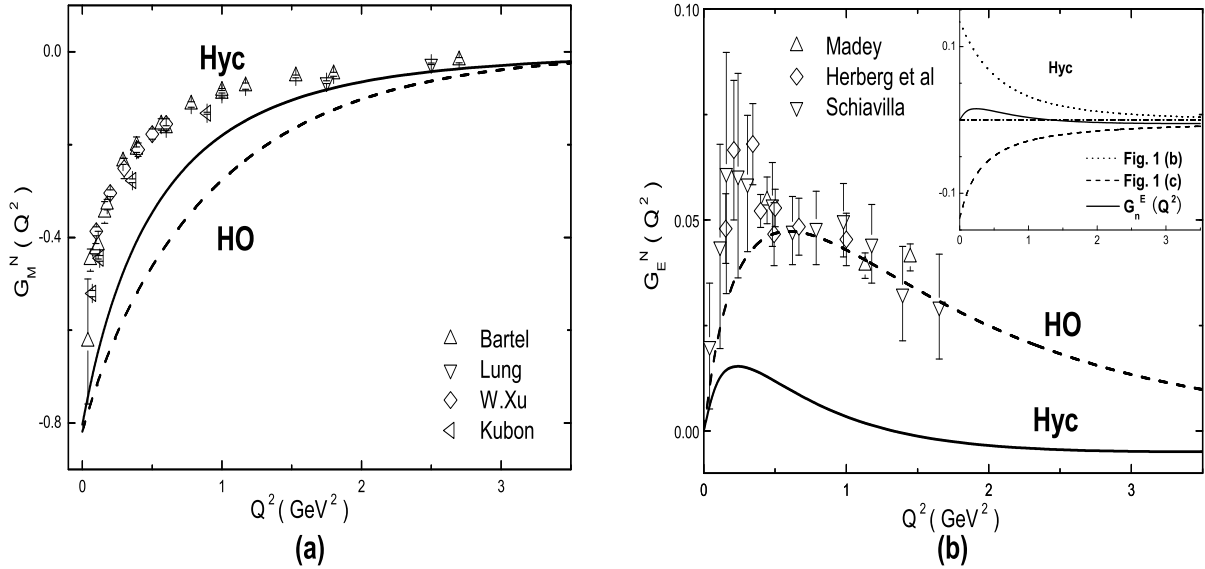


Fig. 3: Neutron magnetic (a) and electric (b) form factors predicted by the Hyc potential and the HO potentials compared with the experimental data [13,14]. The inset in the top right corner of (b) shows the different contributions to G_E^N of Fig. 1 in the Hyc case

order to get a better result in comparison with the experimental data

Figures 4 (a) and 4(b) show the real part of the individual contribution to the helicity amplitudes $A_{3/2}(Q^2)$ and $A_{1/2}(Q^2)$ of the $\Delta(1232)$ resonance respectively. As $Q^2 = 0$, the Hyc prediction of $A_{3/2}$ ($A_{1/2}$) is $-243 \times 10^{-3} \text{ GeV}^{-1/2}$ ($-135 \times 10^{-3} \text{ GeV}^{-1/2}$). Comparing it to the result $A_{3/2} = -187 \times 10^{-3} \text{ GeV}^{-1/2}$ ($A_{1/2} = -108 \times 10^{-3} \text{ GeV}^{-1/2}$) without the pion meson cloud, we clearly see that the results with the pion meson cloud effects reproduce the experimental data $(-250 \pm 8) \times 10^{-3} \text{ GeV}^{-1/2}$ ($(-135 \pm 6) \times 10^{-3} \text{ GeV}^{-1/2}$) [15] much better. It is clear that the contributions from the pion cloud are significant in the real photon point.

Another important observable in the $N - \Delta$ transition is the ratio of $E2/M1$. Our result for it is -0.012. It reasonably agrees with the experimental value reported in Particle data group -0.015 ± 0.004 [15]. If no meson cloud is considered, the ratio of the simple hCQM (without the configuration mixing) vanishes. Our results show that the meson cloud effect also plays a role on this ratio. Thus, we conclude that we are able to reproduce, at least partially, the experimental data of the $N - \Delta$ transition in the low Q^2 region with hCQM and with the pion meson cloud.

5 Summary

In this work, we have studied the nucleon form factors and the form factors of $\gamma^* N \rightarrow \Delta$ transitions based on a hCQM with Hyc wave function. The pion meson cloud effects are explicitly included and discussed. From our numerical results, one may conclude that the pion cloud contribution is crucial for a reasonable explanation of the measured nucleon form factors as well as of the helicity amplitudes of the $\Delta(1232)$ resonance.

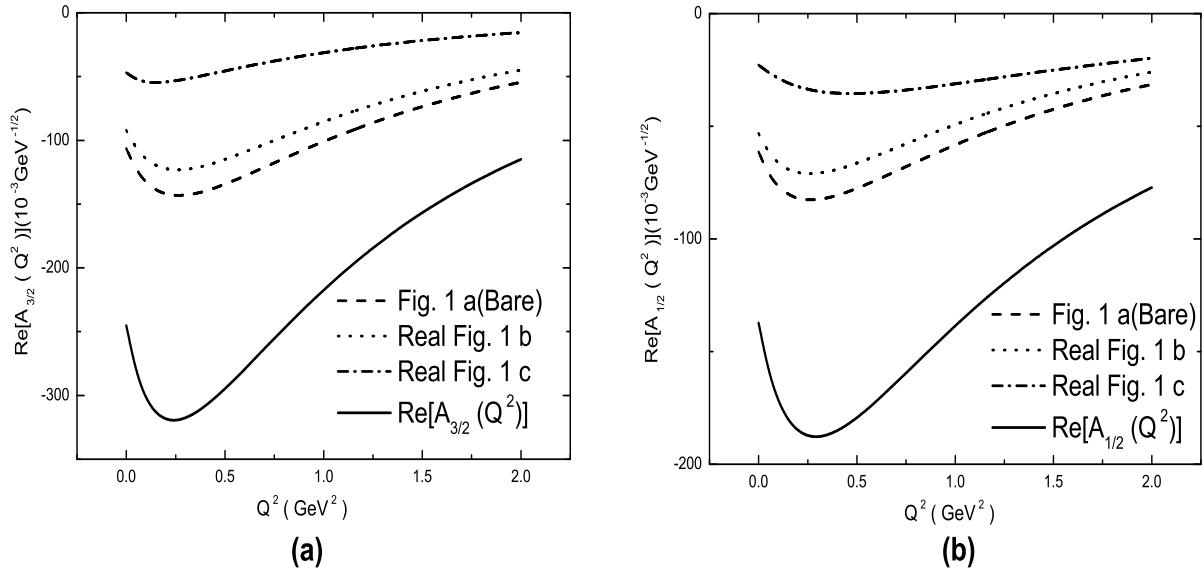


Fig. 4: The real part of the helicity amplitudes (a) $A_{3/2}$ and (b) $A_{1/2}$ (b) of the $\Delta(1232)$ resonance with the different contributions of Fig. 1.

Certainly, the pion meson cloud mainly affects the observables in the low momentum transfer region. In the large momentum transfer region, we need the hypercentral constituent quark model in the relativized version [16], in addition to the inclusion of a perturbative pion cloud. It will also be important to explore the effect of $SU(6)$ violating admixtures in the baryon wave functions [1] simultaneously, since the deformations of the nucleon and Δ wave functions can also provide part of the E2/M1 ratio and part of the neutron charge form factor.

Finally, from the analysis of our results, we see that a hypercentral constituent quark potential model (with a hyper-coulomb plus a linear confinement term and the hyperfine term) together with the pion cloud corrections might be able to give a more reasonable description of the form factors of the nucleon and of the transition form factors of the $\Delta(1232)$. It is of great interest to see the pion meson cloud effects on the Roper resonance and on the resonances $S_{11}(1535)$ and $D_{13}(1520)$ since the transition amplitude to the $S_{11}(1535)$ resonance can be well explained by the Hypercentral Constituent Quark Model in a large $Q^2 \sim 1 \text{ GeV}^2$ region. This work is in progress.

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